Starter Question

The straight line l passes through the points with coordinates $(-4,9)$ and $(4,-3)$.

a) Find an equation for l, giving the answer in the form $ax+by=c$ where a, b and c are integers.

The straight line l meets the coordinate axes at the points A and B.

b) Determine the area of the triangle OAB, where O is the origin.

Starter Question

(a) GRADIENT $= \frac{y_2-y_1}{x_2-x_1} = \frac{-3-9}{4+4} = \frac{-12}{8} = -\frac{3}{2}$

USING $(-4, 9)$

$y - y_0 = m(x - x_0)$

$y - 9 = -\frac{3}{2}(x+4)$

$2y - 18 = -3x - 12$

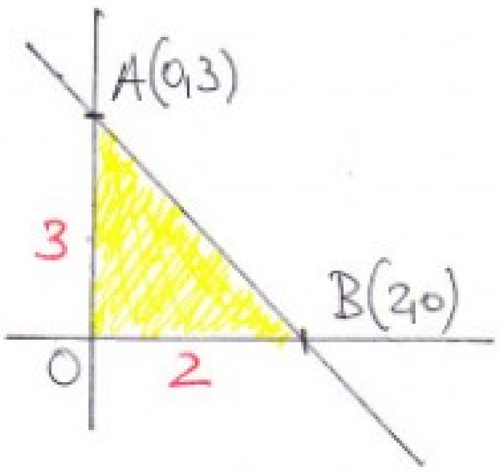
$2y + 3x = 6$

(b) when $x=0$ $2y=6$

$y=3$

when $y=0$ $3x=6$

$x=2$



AREA $= \frac{1}{2} \times 2 \times 3 = 3$

H Integration

H3

Evaluate definite integrals; use a definite integral to find the area under a curve and the area between two curves.

Students should be able to:

- understand and use the fact that for a function, f, where $f(x) \geq 0$ for $a \leq x \leq b$ the area between the x-axis, the curve $y = f(x)$ and the lines $x = a$ and $x = b$ is given by

$$\text{area} = \int_a^b f(x)\,dx$$

- understand that for areas lying **below** the x-axis the definite integral will give the negative of the required value
- find areas between curves and straight lines.

Notes

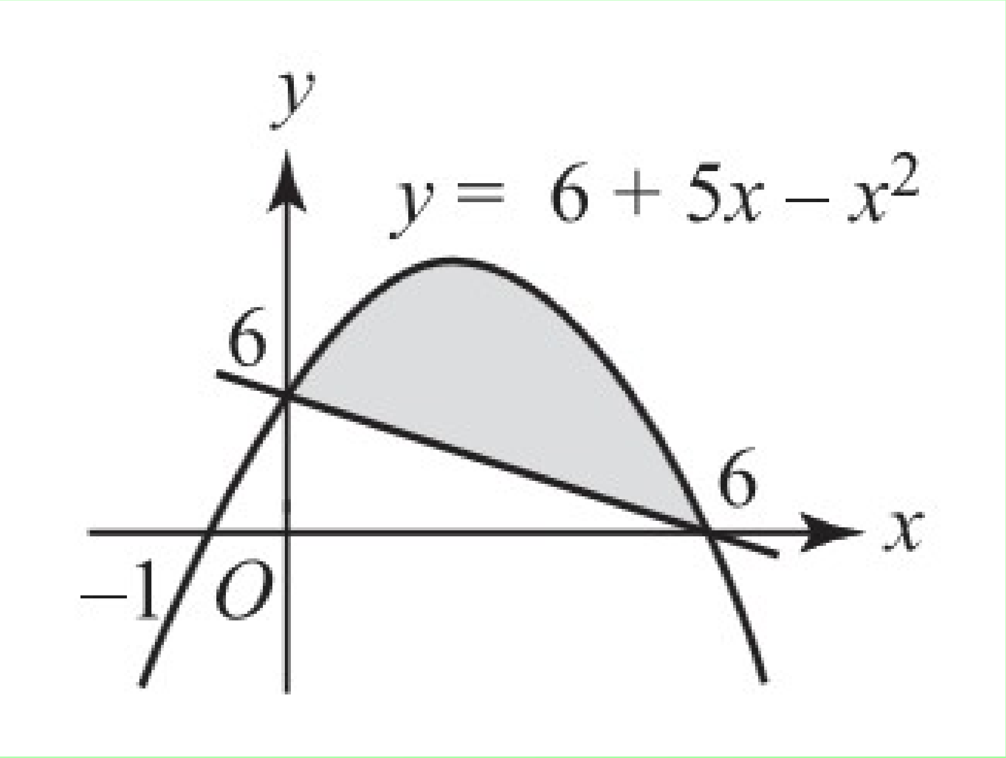
- Definite integrals can be found on a calculator and students are expected to do this in exams. If exact answers are required, these will usually require a non-calculator method.
- Students are **not** expected to find an area between a curve and the y-axis, by integrating an expression for x with respect to y

Tuesday, May 27, 2025

4.7 Area under a curve problem solving

Example 1: Find the shaded area

Area under the curve:



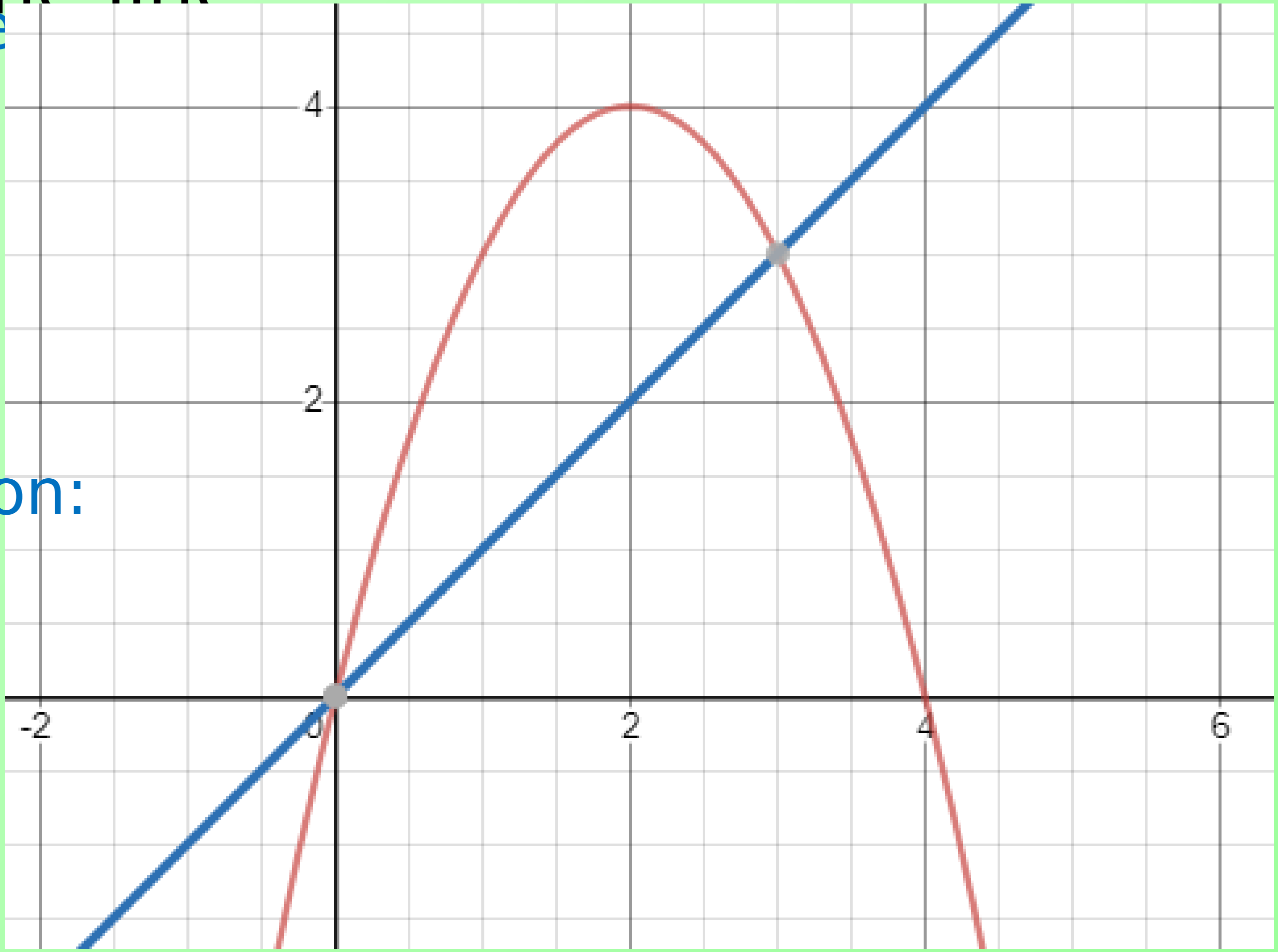
Area of triangle:

Shaded area:
square units

Example 2 Sketch the curve .
Find the area of the region bounded by the curve
and the line

To sketch the curve
-intercepts
-intercept
Shape: negative quadratic

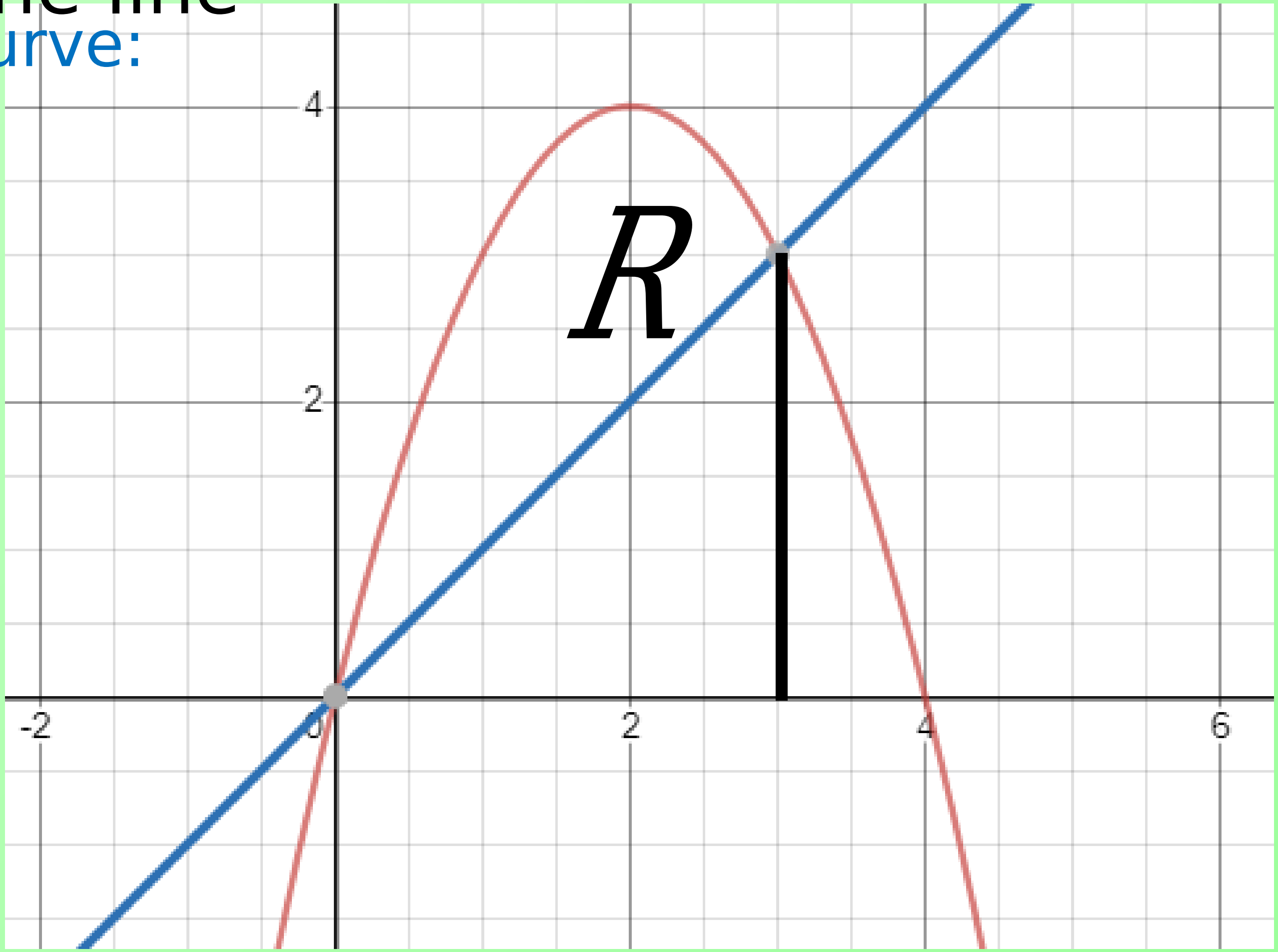
Points of intersection:



So we integrate from 0 to 3

Example 2 Sketch the curve .
Find the area of the region bounded by the curve
 and the line

Area under the curve:



Area of triangle:

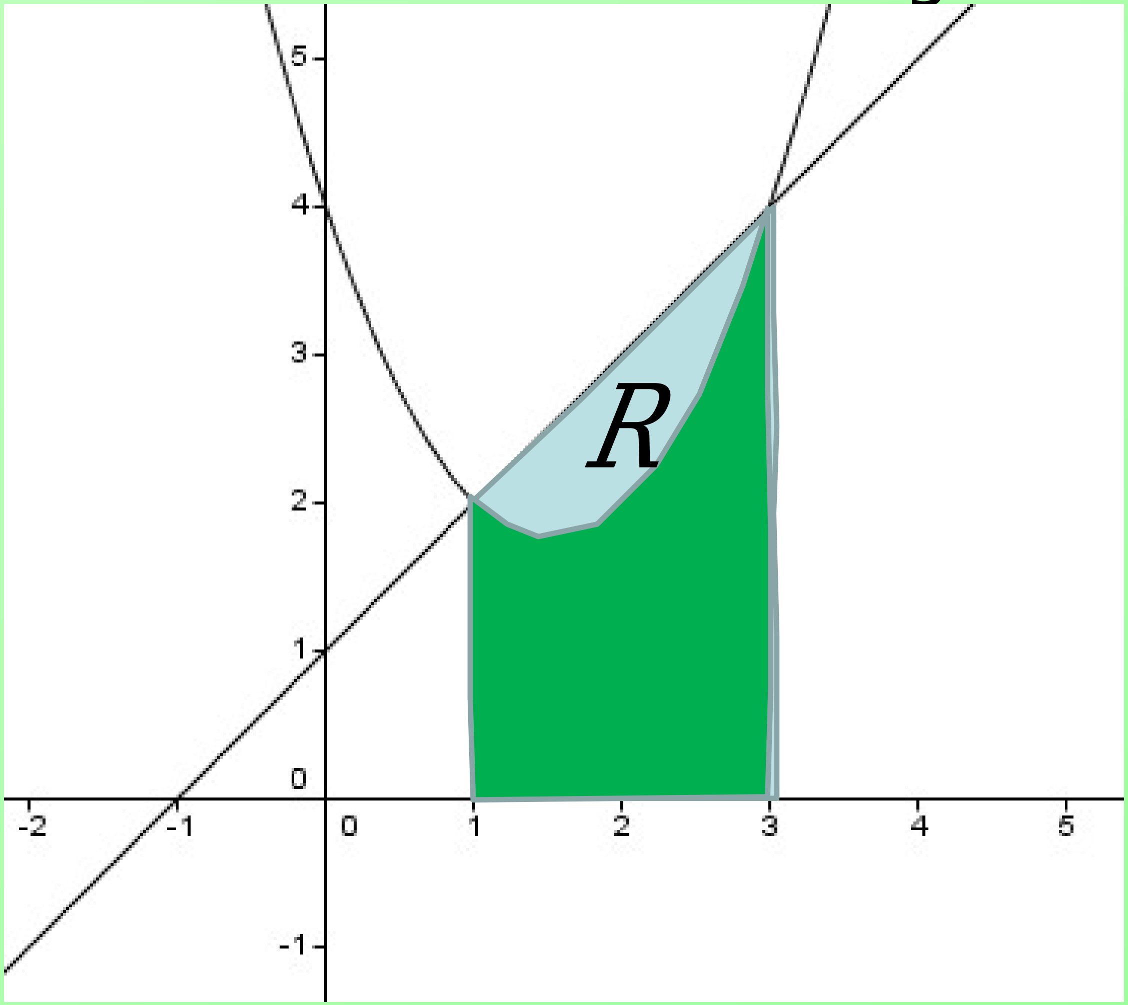
 square units

Example 3 Find the points of intersection of the curve

and the line .

Calculate the area of the finite region bounded

Points of intersection:



So we integrate from 1 to 3

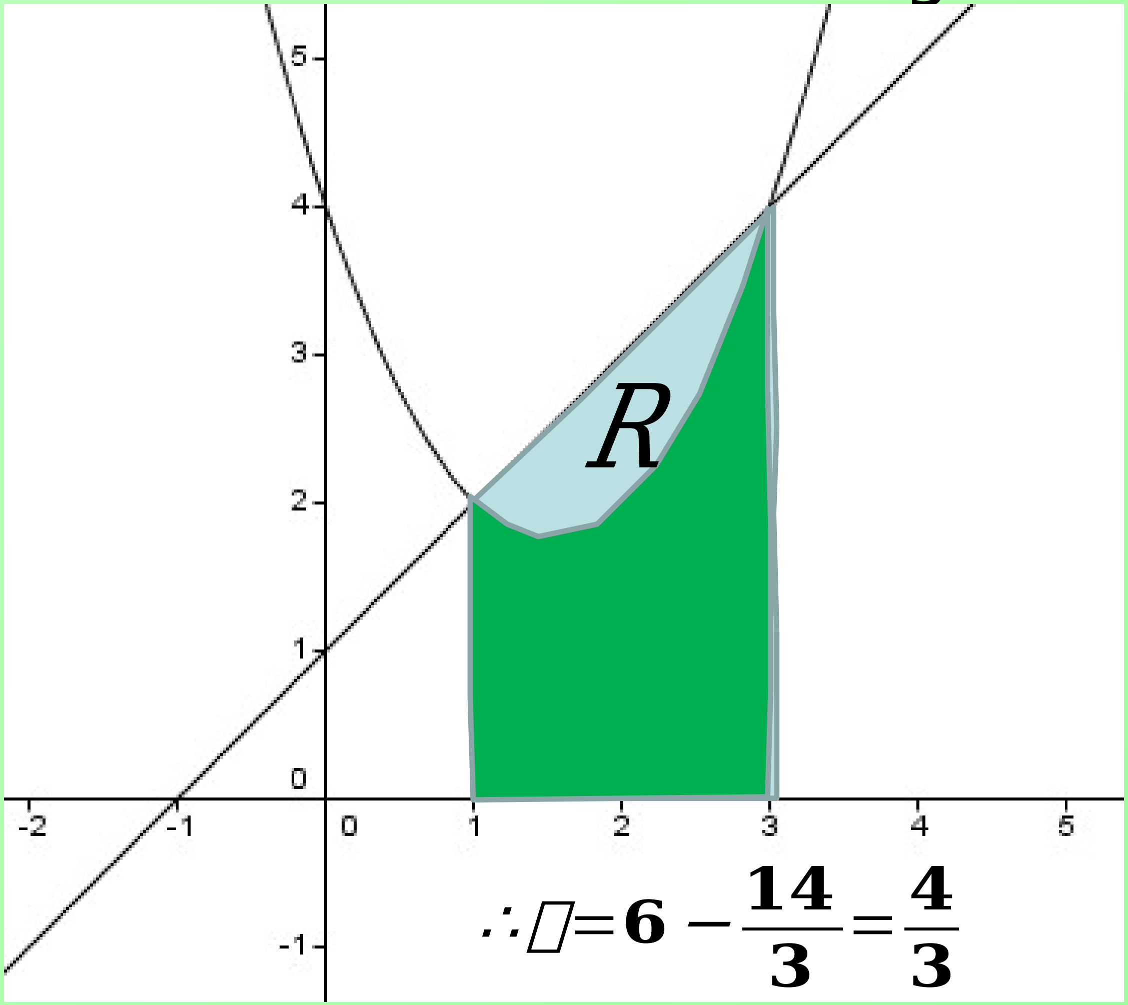
trapezium area – area under the curve

Example 3 Find the points of intersection of the curve
and the line .
Calculate the area of the finite region bounded

trapezium area − area under the curve

Trapezium area:

Area under curve:



$$\therefore \square = 6 - \frac{14}{3} = \frac{4}{3}$$

4.7 Area under a curve problem solving

Areas worksheet

Challenge

Given that $\int_{k}^{3k} \frac{3x+2}{8}\,dx = 7$ and $k > 0$, calculate the value of k.

The point P lies on the curve with equation $y = x^2$, $x > 0$.

The finite region bounded by the curve, the tangent to the curve at P and the y axis has area of 72 square units.

Determine the x coordinate of P.

Challenge

Answer

$$\int_k^{3k} \frac{3x+2}{8}\,\mathrm{d}x = 7$$

$$\int_k^{3k} \left(\frac{3x}{8} + \frac{1}{4}\right)\mathrm{d}x = 7$$

$$\left[\frac{1}{2}\frac{3x^2}{8} + \frac{x}{4}\right]_k^{3k} = 7$$

$$\left[\frac{3x^2}{16} + \frac{x}{4}\right]_k^{3k} = 7$$

$$\left(\frac{3(3k)^2}{16} + \frac{(3k)}{4}\right) - \left(\frac{3k^2}{16} + \frac{k}{4}\right) = 7$$

$$\left(\frac{27k^2}{16} + \frac{3k}{4}\right) - \left(\frac{3k^2}{16} + \frac{k}{4}\right) = 7$$

$$\frac{24k^2}{16} + \frac{k}{2} = 7$$

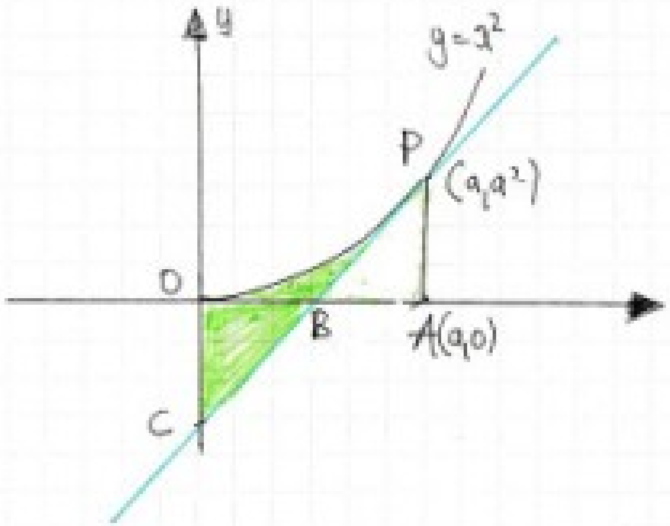
$$24k^2 + 8k - 112 = 0$$

$$3k^2 + k - 14 = 0$$

$$(3k+7)(k-2) = 0$$

$$k = -\tfrac{7}{3} \text{ or } k = 2$$

As $k > 0$, $k = 2$



$y = x^2$

$\frac{dy}{dx} = 2x$

$\left.\frac{dy}{dx}\right|_{x=a} = 2a$

EQUATION OF THE TANGENT AT $P(a, a^2)$

$y - a^2 = 2a(x - a)$

$y - a^2 = 2ax - 2a^2$

$y = 2ax - a^2$

WHEN $x = 0$ $y = -a^2$ $\Longrightarrow$ $C(0, -a^2)$

WHEN $y = 0$ $x = \frac{a}{2}$ $\Longrightarrow$ $B(\frac{a}{2}, 0)$

HENCE THE TRIANGLES $\hat{OBC}$ & $\hat{BAP}$ ARE IDENTICAL

SO THE REQUIRED AREA IS THE SAME AS THE INTEGRAL

$$\int_0^a x^2\,dx = \frac{1}{3}a^3$$

SO $\frac{1}{3}a^3 = 72$

$a^3 = 3 \times 72$

$a^3 = 3 \times (3 \times 3 \times 2 \times 2 \times 2)$

$a^3 = (3 \times 3 \times 3) \times (2 \times 2 \times 2)$

$a = 6$